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PORE EXPANSION IN PLASTIC METALS UNDER SPALL

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The necessity to construct adequate models of material rupture under intensive dynamic loads of shock-wave nature requires a more complete comprehension of the regularities of generation and growth of individual damage during rupture. It is shown [1] that in the case of plastic metals such as aluminum and copper the damage being formed during spall is in the form of pores whose shape is almost spherical. On the basis of experimental investigations, an empirical regularity is proposed in [1] that describes the growth of an individual pore, the so-called law of viscous growth. Estimates are made in [2] for pores expanding in a plastic medium, while a kinematic model based on dislocation mechanics is proposed in [3] to describe pore growth. The model of a viscoplastic medium is used in [4] to model the spalling rupture of copper, where it is shown that satisfactory agreement between the results of experiment and computation is achieved for an extremely low value of the viscosity.

An experimental investigation of the spalling rupture of a number of metals in a broad temperature range was performed in [5]. Results of a metallographic analysis of the tested specimens, presented in [6] and subsequent papers, showed that if the viscous nature of the spalling rupture is inherent for plastic metals with fcc lattice in the whole temperature range investigated, then the viscous nature of rupture is observed under elevated test temperature conditions for metals with other types of crystalline structure. For example, characteristic spall damage in certain metals is presented in Fig. 1: a) lead, T = 0°C, P = 0.69 GPa, ×200; b) nickel, T = 0°C, P = 3.14 GPa, ×800; c) titanium alloy, VT14, T = 800°C, P = 4.25 GPa, ×500; d) Armco iron, T = 800°C, P = 2.74 GPa, ×500.

The behavior of plastic metals under intensive high-velocity plastic strain conditions is described most correctly within the framework of the model of a viscoplastic medium. In this paper the problem of examining the expansion of an isolated pore in a viscoplastic medium under the effect of a short tension pulse is posed, viz.: determine the influence of the fundamental model parameters (viscosity and yield) on the nature of spherical pore expansion. The comparison of such computed results with the results of an experimental observation of the characteristic dimensions of pores being formed during spall can be the basis for determining the viscosity of metals under spalling rupture conditions.

The problem of pore expansion in a viscoplastic medium can be formulated analogously to the problem of its collapse [7]. At a certain time let a pressure pulse P(t) be applied to the outer surface of a spherical cell of radius b_0 with inner cavity of radius a_0. We consider the material of the medium incompressible; consequently, the subsequent motion of the substance is related uniquely to the expansion of the inner cavity. The equation of motion under radial symmetry conditions has the form

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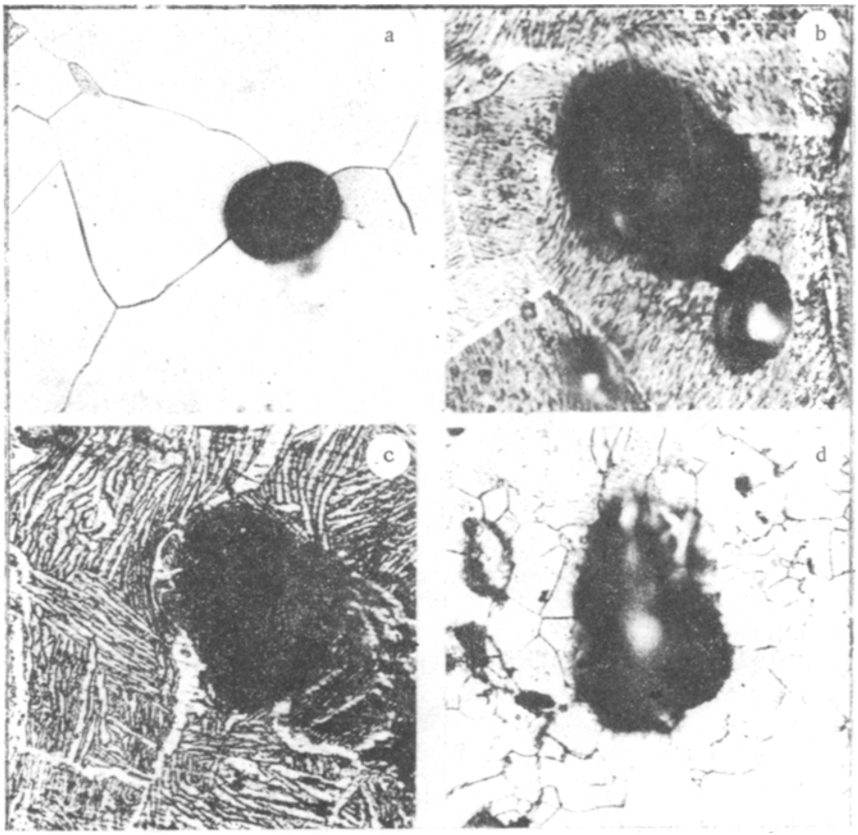

Fig. 1

$$\rho(\partial v/\partial t + v\partial v/\partial x) = \partial\sigma_x/\partial x + 2(\sigma_x - \sigma_\varphi)/x, \tag{1}$$

where σ_x and $\sigma_\varphi = \sigma_\theta$ are the stress tensor components, x is the spacing from the center of symmetry, and v is the velocity of radial motion. The condition for continuity of the normal stress tensor components

$$\sigma_x(b, t) = P(t), \quad \sigma_x(a, t) = 0, \tag{2}$$

is satisfied on the outer surface of the spherical cell and the wall of the cavity, where b and a are the running radii of the outer surface of the spherical cell and the inner cavity. the governing equation of a viscoplastic medium is taken in the form

$$\sigma_x - \sigma_\varphi = -Y + 2\eta(\partial v/\partial x - v/x), \tag{3}$$

where Y is the yield point and η is the viscosity coefficient. The velocity field in an incompressible medium is defined completely by the velocity of the cavity wall

$$v = (da/dt)a^2/x^2. \tag{4}$$

Substituting (3) and (4) into (1) and integrating with the boundary conditions (2) taken into account, we obtain the following equation

$$-P(t) = -2Y\ln\frac{b}{a} - \frac{4\eta(b^3 - a^3)}{ab^3}\frac{da}{dt} + \rho\left[\frac{b^4 - a^4}{2b^4}\left(\frac{da}{dt}\right)^2 - \frac{b-a}{b}\left(a\frac{d^2a}{dt^2} + 2\left(\frac{da}{dt}\right)^2\right)\right]. \tag{5}$$

This differential equation with the initial conditions $a(0) = a_0$, $da/dt(0) = 0$ in which b is related to a by the incompressibility condition $b^3 - a^3 = b^3_0 - a^3_0$, describes the change of the cavity radius in time. At the initial instant the velocity is $da/dt = 0$, while the acceleration is $d^2a/dt^2 > 0$ upon satisfaction of the condition $P(t) > P_* = 2Y \ln (b_0/a_0)$. Therefore, the quantity P_* is the effective yield point of a porous plastic medium under conditions of multilateral tension or compression. The pressure pulse acting on the outer surface of the cell is taken to have the rectangular shape $P(t) = P_0[H(t) - H(t - T)]$, where H(t) is the Heaviside unit function, and T is the loading time. For convenience in analyzing the problem we introduce dimensionless variables and parameters

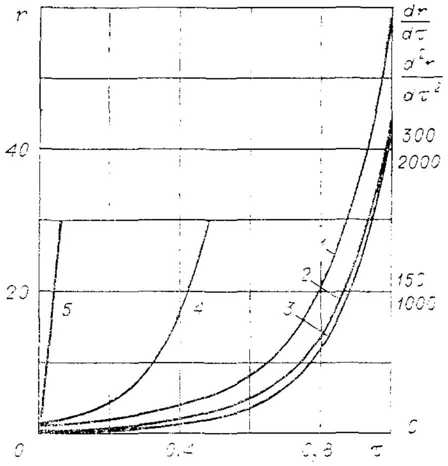

Fig. 2

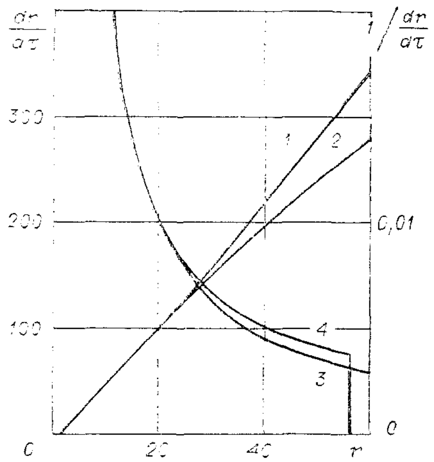

Fig. 3

$$\tau = t/T,\ r = a/a_0,\ \beta = b_0/a_0,\ z = \frac{a}{b} = \frac{r}{\sqrt[3]{\beta^3 + r^3 - 1}},\ p = \frac{P_0 T^2}{\rho a_0^2},\ y = \frac{2YT^2}{\rho a_0^2},\ \mu = \frac{4\eta T}{\rho a_0^2}.$$

Then (5) can be reduced to the form

$$r(1-z)\frac{d^2r}{d\tau^2} = p\left[H(\tau) - H(\tau-1)\right] - y\ln\frac{1}{z} - \frac{\mu}{r}(1-z^3)\frac{dr}{d\tau} - \left[2(1-z) - \frac{1}{2}(1-z^4)\right]\left(\frac{dr}{d\tau}\right)^2. \tag{6}$$

The initial conditions for this equation have the form $r(0) = 1$, $dr/d\tau(0) = 0$.

We consider the solution of the problem formulated in application to the conditions being realized during spalling rupture of plastic metals. To do this, we analyze (6) in application to the results of a specific test (for instance, test 872 from [1]). The minimal radius of the pores being formed is assumed to equal 0.5 μm, the negative pressure in the spall zone is P_0 = 1.12 GPa, and we take the time of elastic wave circulation in the impactor T = 0.75 μsec as the characteristic loading time. It is obtained in [1] that the pore density N with maximal radius 30 μm is $4 \cdot 10^4$ cm^{-3}. These most coarse pores indubitably occur at the very initial loading stage and grow during the whole period T in contrast to the large number of shallow pores that are formed in the later stages of the loading. In this case, the estimate of the spherical cell size yields $b_0 \simeq 1/(2\sqrt[3]{N}) \simeq 150$ μm. Taking the value 0.06 GPa for the yield point of aluminum, we can at once obtain the magnitude of the effective yield point of the material with initial pore distribution under multilateral tension conditions that are similar to those being realized in the spall zone, $P_* = 0.68$ GPa. We also obtain the values of the parameters needed, such as $p = 9.29 \cdot 10^5$, $y = 9.96 \cdot 10^4$, $\beta = 300$. To determine μ, we initially take the value η = 20 Pa•sec used in [1]. In this case, $\mu = 8.89 \cdot 10^4$. The maximal value of r is 60 and the value of z does not exceed 0.2 in this case; consequently, the third and fourth powers of z can be neglected in (6) as compared with one. Therefore, the pore expansion p for the spherical cell under consideration will be described by an equation of the form

$$r\left(1 - \frac{r}{\beta}\right)\frac{d^2r}{d\tau^2} = p - y\ln\frac{\beta}{r} - \frac{\mu}{r}\frac{dr}{d\tau} - \left(\frac{3}{2} - 2\frac{r}{\beta}\right)\left(\frac{dr}{d\tau}\right)^2. \tag{7}$$

The initial short-range and substantially nonlinear stage of pore expansion is characterized for $r \simeq 1$ and $dr/d\tau \simeq 0$ by an extremely large initial value for the acceleration $d^2r/d\tau^2 \simeq p - y\ln\beta = 3.61 \cdot 10^5$, while its subsequent expansion to the radius r = 60 during the time τ = 1 occurs in a quasistationary regime for which the terms containing the acceleration and the square of the velocity are small compared to the other terms in (7).

To consider the initial nonstationary stage of the expansion in (7), we neglect the last inertial term, the difference of r from 1, as well as the quantity r/β as compared with one. In this case (7) becomes

$$d^2r/d\tau^2 + \mu dr/d\tau = p - y\ln\beta.$$

Integrating, we obtain for the pore velocity and radius

$$\frac{dr}{d\tau}=\frac{p-\ln\beta}{\mu}(1-e^{-\mu\tau}),\quad r=1+\frac{p-\ln\beta}{\mu}\tau-\frac{p-\ln\beta}{\mu^2}(1-e^{-\mu\tau})$$

It is seen from the solution obtained that the passage to the quasistationary regime of expansion occurs by a kind of transient with the time constant $1/\mu \simeq 10^{-5}$, while the initial value of the pore velocity in the quasistationary stage equals $(p - y \ln \beta)/\mu \simeq 4$.

In the next quasistationary stage we legitimately neglect both the inertial term and that containing the acceleration in (7). This results in a first-order equation to describe the process of pore expansion while the assumptions taken remain correct:

$$\frac{dr}{d\tau}=\frac{p-y\ln(\beta/r)}{\mu}r. \tag{8}$$

Integrating, we obtain

$$r(\tau)=\exp\left[\frac{p-y\ln\beta}{y}\left(\exp\left(\frac{y}{\mu}\tau\right)-1\right)\right]. \tag{9}$$

Differentiating (9) twice, we also obtain an expression for the pore velocity and acceleration in this stage of the expansion

$$\frac{dr}{d\tau}(\tau)=\frac{p-y\ln\beta}{\mu}\exp\left(\frac{y}{\mu}\tau\right)r(\tau); \tag{10}$$

$$\frac{d^2r}{d\tau^2}(\tau)=\frac{y+(p-y\ln\beta)\exp\left(\frac{y}{\mu}\tau\right)}{\mu}\frac{dr}{d\tau}(\tau). \tag{11}$$

Substituting the values of the known parameters and the value $r = 60$ for $\tau = 1$ into (9), we determine the value $\mu = 1.32 \cdot 10^5$ satisfying these data. The dependences (9)-(11) obtained are presented in Fig. 2, where curves 1-3 correspond to them. A more exact expression can be obtained for the first integral of (7) with the terms initially neglected taken into account. An analysis performed for the pore expansion in the quasistationary stage showed that the terms being neglected are close in magnitude, i.e., $rd^2r/d\tau^2 \simeq (dr/d\tau)^2$. Therefore, the first integral of (7) can be represented more correctly by the formula

$$\frac{dr}{d\tau}=-\frac{\mu}{4r}+\sqrt{\left(\frac{\mu}{4r}\right)^2+\frac{1}{2}\left(p-y\ln\frac{\beta}{r}\right)}. \tag{12}$$

The integral curves 1, 2 corresponding to (8) and (12) are presented in Fig. 3. They show that the divergence starts for $r \simeq 20$, which corresponds to $\tau \simeq 0.8$. Equation (12) is not solved analytically, but an example is presented in Fig. 3 for an elementary graphical integration of the expression $\int_1^{r_1}\frac{1}{\frac{dr}{d\tau}(r)}dr=1$ for cases 1, 2, where r_1 is the pore radius for $\tau = 1$. The integrand curves 3 and 4 should cover an identical area, and taking account of this condition shows that a reduction in the value of r_1 to not more than 56 occurs.

For $\tau > 1$ a certain further inertial expansion of the pores still occurs. In this case (6) with the powers of z to be neglected taken into account will have the form

$$r\left(1-\frac{r}{\beta}\right)\frac{d^2r}{d\tau^2}=-y\ln\frac{\beta}{r}-\frac{\mu}{r}\frac{dr}{d\tau}-\left(\frac{3}{2}-2\frac{r}{\beta}\right)\left(\frac{dr}{d\tau}\right)^2.$$

We obtain the initial conditions from (9) and (10) for $\tau = 1$. The last term in the equation is both the smallest in magnitude and the most progressively diminishing as $dr/d\tau$ is diminished. Also taking into account that the value of r should not change substantially, we estimate the maximal pore radius r_2 by integrating the linear equation

$$r_1\left(1-\frac{r_1}{\beta}\right)\frac{d^2r}{d\tau^2}=-y\ln\frac{\beta}{r_1}-\frac{\mu}{r_1}\frac{dr}{d\tau}.$$

The solution of this equation shows that the pore remains at the radius $r_2 \simeq 65$ up to the time $\tau \simeq 1.04$.

Therefore, the estimates made show that in a specific very typical case (9) describes the nature of the isolated pore expansion in a plastic metal being modeled by a viscoplastic medium, with sufficient correctness. The effect of the reduction in the magnitude of r, associated with taking account of small terms in (7), turns out to be insignificant and compensated, with a high degree of accuracy, by the effect of an increase in r in the retardation stage.

It is interesting to note the separate influence of the strength y and viscosity μ parameters on the nature of the pore expansion. In the case of purely viscous resistance to pore expansion, i.e., for y = 0, we obtain for the quasistationary expansion stage by integrating (8)

$$r = e^{(p/\mu)\tau}.$$

This dependence is shown in Fig. 2 by curve 4 within the limits in which it still remains real. For high values the examination of pore expansion can be performed by a method analogous to that used to obtain the dependence (12). The effective value of the parameter μ needed to describe the experimentally observable pore dimension is $2.27 \cdot 10^5$, and taking account of the pore retardation stage will result in a still more substantial increase in μ. In the case of no viscous resistance to pore expansion, i.e., for μ = 0, Eq. (7) can be integrated under the condition of neglecting the quantity r/β as compared with one, which yields

$$\left(\frac{dr}{d\tau}\right)^2 = \frac{2}{3}(p - y\ln\beta)\left(1 - \frac{1}{r^3}\right) + \frac{2}{3}y\left[\ln r - \frac{1}{3}\left(1 - \frac{1}{r^3}\right)\right]$$

Within the limits of variation of r from 2 to 30, an increase in the velocity occurs from the value $dr/d\tau \simeq (2/3)(p - y \ln \beta)$ of not more than 1.5 times. Consequently, a pore expansion trajectory can be constructed approximately under the conditions of ideal plasticity by using the constant value presented for the velocity. The dependence r(τ) constructed in this manner, which is its lower bound as r increases, is shown by line 5 in Fig. 2.

An estimate of the rise in temperature of the material on the inner surface of the expanding pore can be made in the same way. For instance, in the case of pure plasticity $\Delta T = 2Y \ln\left(\frac{a_1}{a_0}\right)\Big/\rho c \simeq 200°K$, where a_1 is the maximum pore radius, and c is the specific heat. Taking account of viscous dissipation results in the fact that the surface temperature can exceed the melting point.

One of the important and still unclarified questions remains the question of the minimal size of the microscopic pores being formed. In [8], which is a continuation of the investigation started in [1], it is mentioned that microdamage is observed for a magnification of ×100. Hence, the value of the minimal pore radius of 1 μm is apparently taken quite conditionally. Observation of the nature of spalling rupture in plastic metals investigated in [5] and subsequent papers displayed the presence of pores with a minimal radius of about 0.5 μm for a magnification of ×1000. This value is close to the limit of confident resolution of the optical microscope, although pores of minimal dimension can still be distinguished sufficiently clearly from microscopic inclusions at this magnification. One of the sources of pores being formed is apparently the exposure of microcracks generated at stressed bulk dislocation clusters such as dislocation cells and balls. The dimensions of these microcracks attain the dimensions of dislocation clusters [9]. A cellular dislocation structure is formed earlier in metals not subjected to preliminary annealing because of the preceding plastic strain during the technological processing. The dislocation structure occurring in such metals as nickel and copper under shockwave loading is characterized by dislocation cell dimensions of 0.15-1.4 μm [10]. The fundamental source of the pores being formed together with the structural inhomogeneities are apparently particles of foreign impurities and inclusions that are present in large quantities in technical metals and especially in alloys.

The expression (9) obtained for the radius of the expanding pore in the case of smallness of the ratio y/μ as compared with one or in the case of small τ goes over into the expression

$$r(\tau) = \exp\left(\frac{p - y\ln\beta}{\mu}\tau\right), \tag{13}$$

which corresponds exactly to that obtained in [1] on the basis of the so-called law of viscous growth. The meaning of the concept used in [1] of the limit of pore growth σg_0 which is none other than the yield point of the medium with initial distribution of minimal size pores under multilateral tension conditions, becomes comprehensible.

To develop viscous spalling rupture in plastic metals, it is necessary that the negative pressure applied exceed the yield point of the porous medium with initial distribution of minimal size pores $P_* = 2Y \times \ln \frac{1}{2\sqrt[3]{N}\, a_0}$. Analysis of the results of [1, 8] as well as a metallographic study of the specimens tested in [5] yield the characteristic values 0.5 μm and 10^4 cm^{-3} for a_0 and N. Therefore, the negative pressure in the spall zone should exceed the value of P_*, which is 12.3Y in this case. Variation of the quantity N within the limits 10^3-10^5 results in a change in P_* within the limits (13.8-10.8)Y. The loading level corresponding to the generation of micropores in a sufficient quantity can be above the level of P_* and manifests an explicit time dependence (see [11], for example).

We now examine the possibility of determining the viscosity of metals under conditions corresponding to viscous spalling rupture due to pore growth. On the basis of (13), an upper bound can be found at once for the parameter $\mu = p - y \ln \beta$. In the case of aluminum the upper bound is $\mu = 3.61 \cdot 10^5$ which yields the upper bound for the magnitude of the viscosity η = 81.2 Pa•sec. The value $\mu = 1.32 \cdot 10^5$ obtained on the basis of an analysis of individual pore expansion corresponds to the viscosity of aluminum η = 30 Pa•sec. The magnitude of the viscosity in certain other metals can be estimated in a similar manner. To do this we solve (9) for η by substituting the necessary parameters and reducing to dimensional form

$$\eta = \frac{2YT}{4\ln\left(1 + \dfrac{2Y \ln \dfrac{a_1}{a_0}}{P_0 - 2Y \ln \dfrac{b_0}{a_0}}\right)}.$$

We illustrate the possibility of determining the quantity η by several examples. For copper (test S24 from [8]), for P_0 = 1.8 GPa, Y = 0.06 GPa, T = 0.25 μsec, a_0 = 0.5 μm, a_1 = 45 μm, $N = 2 \cdot 10^4$ cm^{-3} we obtain η = 20 Pa•sec. For lead with P_0 = 0.6 GPa, Y = 0.01 GPa, T = 1.3 μsec, a_0 = 0.5 μm, a_1 = 50 μm, $N \simeq 10^5$ cm^{-3} we obtain η = 38 Pa•sec. For nickel with P_0 = 3.14 GPa, Y = 0.2 GPa, T = 1.3 μsec, a_0 = 0.5 μm, a_1 = 20 μm, $N \simeq 10^4$ cm^{-3} we obtain η = 113 Pa•sec. For the titanium alloy VT14 at a 800°C temperature and with P_0 = 3.6 GPa, Y = 0.1 GPa, a_0 = 0.5-1 μm, a_1 = 60 μm, $N \simeq 10^4$ cm^{-3} we obtain η = 96-230 Pa•sec, depending on the value of a_0. The value a_0 = 1 μm is taken since it is difficult to observe pores with radius less than 1 μm in a specific structure of the alloy VT14 (see Fig. 1c), and the value a_0 = 0.5 μm is common for a whole series of previous materials and apparently more acceptable in the interests of a comparative analysis.

The values obtained for the metal viscosity compare well with results obtained for other test conditions. The most satisfactory agreement between experimental and theoretical results is observed in [12] for η = 20 Pa•sec for the shock-wave compression of porous aluminum. The upper bounds for the viscosity of aluminum and copper, obtained from measuring the profile of strong shock fronts, are 100 and 300 Pa•sec in [13]. Estimates of the strain rate $\dot{\varepsilon}$ made for these conditions yield ~10^8 sec^{-1}, while estimates of the mobile dislocation density ρ_d yield ~10^9 cm^{-2}. Under pore expansion conditions the estimates of $\dot{\varepsilon}$ yield ~10^7 sec^{-1}. Estimates can also be made for the mobile dislocation density $\rho_d = B/\eta b^2$, where B is the stagnation factor and b is the Burgers vector. The mean value of B for metals with fcc lattice is ~$4 \cdot 10^{-5}$ Pa•sec [14], which yields ~10^9 cm^{-2} for the estimate of ρ_d.

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BENDING OF AN ANISOTROPIC PLATE CONTAINING AN ANISOTROPIC ELASTIC INCLUSION

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A thin plate of thickness h is considered that has a curvilinear hole into which is soldered an elastic body made of another material. The plate and the inclusion have rectilinear anisotropy with respect to the elastic properties of the material and at each point have a plane of elastic symmetry parallel to the median plane xOy. The principal elasticity directions for the plate and inclusion are at an angle φ (Fig. 1). The line L dividing the regions $S^{(1)}$ and $S^{(2)}$ corresponding to the different anisotropic materials is described by an equation of the form

$$t = x + iy = R\left(e^{i\theta} + \sum_{k=1}^{N} C_k e^{-ik\theta}\right), \quad \sum_{k=1}^{N} k\,|C_k|^2 < 1. \tag{1}$$

Along line L between regions $S^{(\alpha)}$ (α = 1, 2), the conjugation conditions should apply:

$$M_n^{(1)} = M_n^{(2)}, \; N_n^{(1)} + \frac{\partial H_{n\tau}^{(1)}}{\partial s} = N_n^{(2)} + \frac{\partial H_{n\tau}^{(2)}}{\partial s}, \tag{2}$$
$$W^{(1)} = W^{(2)}, \; \frac{\partial W^{(1)}}{\partial n} = \frac{\partial W^{(2)}}{\partial n},$$

while in parts of the plate remote from the inclusion the bending and torsional moments are bounded: $M^{\infty}_x = M_1$, $M^{\infty}_y = M_2$, $M^{\infty}_{xy} = H_{12}$. There are no external localized forces and distributed loads normal to the median plane in the regions $S^{(\alpha)}$ (α = 1, 2). Here n and τ are the normal and tangent to line L.

In the analytic solution, the region $S^{(1)}$ will be considered as infinite (the perturbation in the elastic state of the plate due to the inclusion does not attain the outer boundary of the plate).

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